

Team Note of Deobureo Minkyu Party

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Compiled on October 9, 2018

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ALL BELOW HERE ARE USELESS IF YOU READ THE STATEMENT WRONG

1 Quote

렉스를 렉스답게 든든한 연습헬팟 더불어민규당

hyea: Prove by solving.

koosaga: Locality to the rescue!

alex9801: Isshoman Beenzino

2 Flows, Matching

2.1 Hopcroft-Karp Bipartite Matching

```
const int MAXN = 50005, MAXM = 50005;
vector<int> gph[MAXN];
int dis[MAXN], l[MAXN], r[MAXN], vis[MAXN];
void clear(){ for(int i=0; i<MAXN; i++) gph[i].clear(); }
void add_edge(int l, int r){ gph[l].push_back(r); }
bool bfs(int n){
    queue<int> que;
    bool ok = 0;
    memset(dis, 0, sizeof(dis));
    for(int i=0; i<n; i++){
        if(l[i] == -1 && !dis[i]){
            que.push(i);
            dis[i] = 1;
        }
    }
    while(!que.empty()){
        int x = que.front();
        que.pop();
        for(auto &i : gph[x]){
            if(r[i] == -1) ok = 1;
            else if(!dis[r[i]]){
                dis[r[i]] = dis[x] + 1;
                que.push(r[i]);
            }
        }
    }
    return ok;
}
bool dfs(int x){
    if(vis[x]) return 0;
    vis[x] = 1;
    for(auto &i : gph[x]){
        if(r[i] == -1 || (!vis[r[i]] && dis[r[i]] == dis[x] + 1 && dfs(r[i]))){
            l[x] = i; r[i] = x;
            return 1;
        }
    }
    return 0;
}
int match(int n){
```

```
memset(l, -1, sizeof(l));
memset(r, -1, sizeof(r));
int ret = 0;
while(bfs(n)){
    memset(vis, 0, sizeof(vis));
    for(int i=0; i<n; i++) if(l[i] == -1 && dfs(i)) ret++;
}
return ret;
}
bool chk[MAXN + MAXM];
void rdfs(int x, int n){
    if(chk[x]) return;
    chk[x] = 1;
    for(auto &i : gph[x]){
        chk[i + n] = 1;
        rdfs(r[i], n);
    }
}
vector<int> getcover(int n, int m){ // solve min. vertex cover
    match(n);
    memset(chk, 0, sizeof(chk));
    for(int i=0; i<n; i++) if(l[i] == -1) rdfs(i, n);
    vector<int> v;
    for(int i=0; i<n; i++) if(!chk[i]) v.push_back(i);
    for(int i=n; i<n+m; i++) if(chk[i]) v.push_back(i);
    return v;
}
```

2.2 Dinic's Algorithm

```
const int MAXN = 505;
struct edg{ int pos, cap, rev; };
vector<edg> gph[MAXN];
void clear(){ for(int i=0; i<MAXN; i++) gph[i].clear(); }
void add_edge(int s, int e, int x){
    gph[s].push_back({e, x, (int)gph[e].size()});
    gph[e].push_back({s, 0, (int)gph[s].size()-1});
}
int dis[MAXN], pnt[MAXN];
bool bfs(int src, int sink){
    memset(dis, 0, sizeof(dis));
    memset(pnt, 0, sizeof(pnt));
    queue<int> que;
    que.push(src);
    dis[src] = 1;
    while(!que.empty()){
        int x = que.front();
        que.pop();
        for(auto &e : gph[x]){
            if(e.cap > 0 && !dis[e.pos]){
                dis[e.pos] = dis[x] + 1;
                que.push(e.pos);
            }
        }
    }
}
return dis[sink] > 0;
```

```

}
int dfs(int x, int sink, int f){
    if(x == sink) return f;
    for(; pnt[x] < gph[x].size(); pnt[x]++){
        edg e = gph[x][pnt[x]];
        if(e.cap > 0 && dis[e.pos] == dis[x] + 1){
            int w = dfs(e.pos, sink, min(f, e.cap));
            if(w){
                gph[x][pnt[x]].cap -= w;
                gph[e.pos][e.rev].cap += w;
                return w;
            }
        }
    }
    return 0;
}

int match(int src, int sink){
    int ret = 0;
    while(bfs(src, sink)){
        int r;
        while((r = dfs(src, sink, 2e9))) ret += r;
    }
    return ret;
}

```

2.3 Min Cost Max Flow

```

const int MAXN = 100;
struct mincostflow{
    struct edg{ int pos, cap, rev, cost; };
    vector<edg> gph[MAXN];
    void clear(){
        for(int i=0; i<MAXN; i++) gph[i].clear();
    }
    void add_edge(int s, int e, int x, int c){
        gph[s].push_back({e, x, (int)gph[e].size(), c});
        gph[e].push_back({s, 0, (int)gph[s].size()-1, -c});
    }
    int dist[MAXN], pa[MAXN], pe[MAXN];
    bool inque[MAXN];
    bool spfa(int src, int sink){
        memset(dist, 0x3f, sizeof(dist));
        memset(inque, 0, sizeof(inque));
        queue<int> que;
        dist[src] = 0;
        inque[src] = 1;
        que.push(src);
        bool ok = 0;
        while(!que.empty()){
            int x = que.front();
            que.pop();
            if(x == sink) ok = 1;
            inque[x] = 0;
            for(int i=0; i<gph[x].size(); i++){
                edg e = gph[x][i];
                if(e.cap > 0 && dist[e.pos] > dist[x] + e.cost){

```

```

                    dist[e.pos] = dist[x] + e.cost;
                    pa[e.pos] = x;
                    pe[e.pos] = i;
                    if(!inque[e.pos]){
                        inque[e.pos] = 1;
                        que.push(e.pos);
                    }
                }
            }
        }
        return ok;
    }
    int match(int src, int sink){
        int ret = 0;
        while(spfa(src, sink)){
            int cap = 1e9;
            for(int pos = sink; pos != src; pos = pa[pos]){
                cap = min(cap, gph[pa[pos]][pe[pos]].cap);
            }
            ret += dist[sink] * cap;
            for(int pos = sink; pos != src; pos = pa[pos]){
                int rev = gph[pa[pos]][pe[pos]].rev;
                gph[pa[pos]][pe[pos]].cap -= cap;
                gph[pos][rev].cap += cap;
            }
        }
        return ret;
    }
}mcmf;

```

2.4 Hell-Joseon style MCMF

```

const int MAXN = 100;
struct mincostflow{
    struct edg{ int pos, cap, rev, cost; };
    vector<edg> gph[MAXN];
    void clear(){ for(int i=0; i<MAXN; i++) gph[i].clear(); }
    void add_edge(int s, int e, int x, int c){
        gph[s].push_back({e, x, (int)gph[e].size(), c});
        gph[e].push_back({s, 0, (int)gph[s].size()-1, -c});
    }
    int phi[MAXN], inque[MAXN], dist[MAXN];
    void prep(int src, int sink){
        memset(phi, 0x3f, sizeof(phi));
        memset(dist, 0x3f, sizeof(dist));
        queue<int> que;
        que.push(src);
        inque[src] = 1;
        while(!que.empty()){
            int x = que.front();
            que.pop();
            inque[x] = 0;
            for(auto &i : gph[x]){
                if(i.cap > 0 && phi[i.pos] > phi[x] + i.cost){
                    phi[i.pos] = phi[x] + i.cost;
                    if(!inque[i.pos]){

```

```

        inque[i.pos] = 1;
        que.push(i.pos);
    }
}
}
for(int i=0; i<MAXN; i++){
    for(auto &j : gph[i]){
        if(j.cap > 0) j.cost += phi[i] - phi[j.pos];
    }
}
priority_queue<pi, vector<pi>, greater<pi> > pq;
pq.push(pi(0, src));
dist[src] = 0;
while(!pq.empty()){
    auto l = pq.top();
    pq.pop();
    if(dist[l.second] != l.first) continue;
    for(auto &i : gph[l.second]){
        if(i.cap > 0 && dist[i.pos] > l.first + i.cost){
            dist[i.pos] = l.first + i.cost;
            pq.push(pi(dist[i.pos], i.pos));
        }
    }
}
}
bool vis[MAXN];
int ptr[MAXN];
int dfs(int pos, int sink, int flow){
    vis[pos] = 1;
    if(pos == sink) return flow;
    for(; ptr[pos] < gph[pos].size(); ptr[pos]++){
        auto &i = gph[pos][ptr[pos]];
        if(!vis[i.pos] && dist[i.pos] == i.cost + dist[pos] && i.cap > 0){
            int ret = dfs(i.pos, sink, min(i.cap, flow));
            if(ret != 0){
                i.cap -= ret;
                gph[i.pos][i.rev].cap += ret;
                return ret;
            }
        }
    }
    return 0;
}
int match(int src, int sink, int sz){
    prep(src, sink);
    for(int i=0; i<sz; i++) dist[i] += phi[sink] - phi[src];
    int ret = 0;
    while(true){
        memset(ptr, 0, sizeof(ptr));
        memset(vis, 0, sizeof(vis));
        int tmp = 0;
        while((tmp = dfs(src, sink, 1e9))){
            ret += dist[sink] * tmp;
            memset(vis, 0, sizeof(vis));
        }
    }
}

```

```

    tmp = 1e9;
    for(int i=0; i<sz; i++){
        if(!vis[i]) continue;
        for(auto &j : gph[i]){
            if(j.cap > 0 && !vis[j.pos]){
                tmp = min(tmp, (dist[i] + j.cost) - dist[j.pos]);
            }
        }
    }
    if(tmp > 1e9 - 200) break;
    for(int i=0; i<sz; i++){
        if(!vis[i]) dist[i] += tmp;
    }
}
return ret;
}
}mcmf;

```

2.5 Circulation Problem

```

struct circ{
    maxflow mf;
    lint lsum;
    void clear(){
        lsum = 0;
        mf.clear();
    }
    void add_edge(int s, int e, int l, int r){
        lsum += l;
        mf.add_edge(s + 2, e + 2, r - l);
        mf.add_edge(0, e + 2, l);
        mf.add_edge(s + 2, 1, l);
    }
    bool solve(int s, int e){
        mf.add_edge(e+2, s+2, 1e9); // to reduce as maxflow with lower bounds, in circulation
        // problem skip this line
        return lsum == mf.match(0, 1);
        // to get maximum LR flow, run maxflow from s+2 to e+2 again
    }
}circ;

```

2.6 Min Cost Circulation

// Cycle canceling (Dual of successive shortest path)
 // Time complexity is ridiculously high ($F * \max C * nm^2$). But runs reasonably in practice
 ($V = 70$ in 1s)

```

struct cost_circ{
    struct edg{ int pos, cap, rev, cost; };
    vector<edg> gph[MAXN];
    void clear(){ for(int i=0; i<MAXN; i++) gph[i].clear(); }
    void add_edge(int s, int e, int x, int c){
        gph[s].push_back({e, x, (int)gph[e].size(), c});
        gph[e].push_back({s, 0, (int)gph[s].size()-1, -c});
    }
    int dist[MAXN], par[MAXN], pae[MAXN];
}

```

```

int negative_cycle(int n){
    bool mark[MAXN] = {};
    memset(dist, 0, sizeof(dist));
    int upd = -1;
    for(int i=0; i<=n; i++){
        for(int j=0; j<n; j++){
            int idx = 0;
            for(auto &k : gph[j]){
                if(k.cap > 0 && dist[k.pos] > dist[j] + k.cost){
                    dist[k.pos] = dist[j] + k.cost;
                    par[k.pos] = j;
                    pae[k.pos] = idx;
                    if(i == n){
                        upd = j;
                        while(!mark[upd]){
                            mark[upd] = 1;
                            upd = par[upd];
                        }
                        return upd;
                    }
                }
            }
            idx++;
        }
    }
    return -1;
}

int match(int n){
    int rt = -1;
    int ans = 0;
    while(~(rt = negative_cycle(n))){
        bool mark[MAXN] = {};
        vector<pi> cyc;
        while(!mark[rt]){
            cyc.push_back(pi(par[rt], pae[rt]));
            mark[rt] = 1;
            rt = par[rt];
        }
        reverse(cyc.begin(), cyc.end());
        int capv = 1e9;
        for(auto &i : cyc){
            auto e = &gph[i.first][i.second];
            capv = min(capv, e->cap);
        }
        for(auto &i : cyc){
            auto e = &gph[i.first][i.second];
            e->cap -= capv;
            gph[e->pos][e->rev].cap += capv;
            ans += e->cost * capv;
        }
    }
    return ans;
}
}circ;

```

2.7 Gomory-Hu Tree

```

struct edg{ int s, e, x; };
vector<edg> eds;
maxflow mf;
void clear(){ eds.clear(); }
void add_edge(int s, int e, int x){ eds.push_back({s, e, x}); }
bool vis[MAXN];
void dfs(int x){
    if(vis[x]) return;
    vis[x] = 1;
    for(auto &i : mf.gph[x]) if(i.cap > 0) dfs(i.pos);
}

vector<pi> solve(int n){ // i - j cut : i - j minimum edge cost. 0 based.
    vector<pi> ret(n); // if i > 0, stores pair(parent, cost)
    for(int i=1; i<n; i++){
        for(auto &j : eds){
            mf.add_edge(j.s, j.e, j.x);
            mf.add_edge(j.e, j.s, j.x);
        }
        ret[i].first = mf.match(i, ret[i].second);
        memset(vis, 0, sizeof(vis));
        dfs(i);
        for(int j=i+1; j<n; j++){
            if(ret[j].second == ret[i].second && vis[j]){
                ret[j].second = i;
            }
        }
        mf.clear();
    }
    return ret;
}

```

2.8 Blossom Algorithm for General Matching

```

const int MAXN = 2020 + 1;
struct GM { // 1-based Vertex index
    int vis[MAXN], par[MAXN], orig[MAXN], match[MAXN], aux[MAXN], t, N;
    vector<int> conn[MAXN];
    queue<int> Q;
    void addEdge(int u, int v) {
        conn[u].push_back(v); conn[v].push_back(u);
    }
    void init(int n) {
        N = n; t = 0;
        for(int i=0; i<=n; ++i) {
            conn[i].clear();
            match[i] = aux[i] = par[i] = 0;
        }
    }
    void augment(int u, int v) {
        int pv = v, nv;
        do {
            pv = par[pv]; nv = match[pv];
            match[pv] = pv; match[pv] = v;
            v = nv;
        } while(v != u);
    }
};

```

```

    } while(u != pv);
}
int lca(int v, int w) {
    ++t;
    while(true) {
        if(v) {
            if(aux[v] == t) return v; aux[v] = t;
            v = orig[par[match[v]]];
        }
        swap(v, w);
    }
}
void blossom(int v, int w, int a) {
    while(orig[v] != a) {
        par[v] = w; w = match[v];
        if(vis[w] == 1) Q.push(w), vis[w] = 0;
        orig[v] = orig[w] = a;
        v = par[w];
    }
}
bool bfs(int u) {
    fill(vis+1, vis+1+N, -1); iota(orig + 1, orig + N + 1, 1);
    Q = queue<int> (); Q.push(u); vis[u] = 0;
    while(!Q.empty()) {
        int v = Q.front(); Q.pop();
        for(int x: conn[v]) {
            if(vis[x] == -1) {
                par[x] = v; vis[x] = 1;
                if(!match[x]) return augment(u, x), true;
                Q.push(match[x]); vis[match[x]] = 0;
            }
            else if(vis[x] == 0 && orig[v] != orig[x]) {
                int a = lca(orig[v], orig[x]);
                blossom(x, v, a); blossom(v, x, a);
            }
        }
    }
    return false;
}
int Match() {
    int ans = 0;
    // find random matching (not necessary, constant improvement)
    vector<int> V(N+1); iota(V.begin(), V.end(), 1);
    shuffle(V.begin(), V.end(), mt19937(0x94949));
    for(auto x: V) if(!match[x]){
        for(auto y: conn[x]) if(!match[y]) {
            match[x] = y, match[y] = x;
            ++ans; break;
        }
    }
    for(int i=1; i<=N; ++i) if(!match[i] && bfs(i)) ++ans;
    return ans;
}
};

```

2.9 Blossom Algorithm for Weighted General Matching

```

//
// https://github.com/tzupengwang/PECaveros/blob/master/codebook/graph/BorrowedGeneralWeightedMatching
// N^3 (but fast in practice)
struct WeightGraph {
    static const int INF = INT_MAX;
    static const int N = 514;
    struct edge{
        int u,v,w; edge(){}
        edge(int ui,int vi,int wi)
            :u(ui),v(vi),w(wi){}
    };
    int n,n_x;
    edge g[N*2][N*2];
    int lab[N*2];
    int match[N*2],slack[N*2],st[N*2],pa[N*2];
    int flo_from[N*2][N+1],S[N*2],vis[N*2];
    vector<int> flo[N*2];
    queue<int> q;
    int e_delta(const edge &e){
        return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
    }
    void update_slack(int u,int x){
        if(!slack[x] || e_delta(g[u][x])<e_delta(g[slack[x]][x]))slack[x]=u;
    }
    void set_slack(int x){
        slack[x]=0;
        for(int u=1;u<=n;++u)
            if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
                update_slack(u,x);
    }
    void q_push(int x){
        if(x<=n)q.push(x);
        else for(size_t i=0;i<flo[x].size();i++)
            q_push(flo[x][i]);
    }
    void set_st(int x,int b){
        st[x]=b;
        if(x>n)for(size_t i=0;i<flo[x].size();++i)
            set_st(flo[x][i],b);
    }
    int get_pr(int b,int xr){
        int pr=find(flo[b].begin(),flo[b].end(),xr)-flo[b].begin();
        if(pr%2==1){
            reverse(flo[b].begin()+1,flo[b].end());
            return (int)flo[b].size()-pr;
        }else return pr;
    }
    void set_match(int u,int v){
        match[u]=g[u][v].v;
        if(u<=n) return;
        edge e=g[u][v];
        int xr=flo_from[u][e.u],pr=get_pr(u,xr);
        for(int i=0;i<pr;++i)set_match(flo[u][i],flo[u][i^1]);
        set_match(xr,v);
    }
}

```

```

rotate(flo[u].begin(), flo[u].begin()+pr, flo[u].end());
}
void augment(int u, int v){
for(;;){
    int xnv=st[match[u]];
    set_match(u, v);
    if(!xnv) return;
    set_match(xnv, st[pa[xnv]]);
    u=st[pa[xnv]], v=xnv;
}
}
int get_lca(int u, int v){
static int t=0;
for(++t; u||v; swap(u, v)){
    if(u==0) continue;
    if(vis[u]==t) return u;
    vis[u]=t;
    u=st[match[u]];
    if(u) u=st[pa[u]];
}
return 0;
}
void add_blossom(int u, int lca, int v){
int b=n+1;
while(b<=n_x&&st[b]) ++b;
if(b>n_x) ++n_x;
lab[b]=0, S[b]=0;
match[b]=match[lca];
flo[b].clear();
flo[b].push_back(lca);
for(int x=u, y; x!=lca; x=st[pa[y]])
    flo[b].push_back(x), flo[b].push_back(y=st[match[x]]), q_push(y);
reverse(flo[b].begin()+1, flo[b].end());
for(int x=v, y; x!=lca; x=st[pa[y]])
    flo[b].push_back(x), flo[b].push_back(y=st[match[x]]), q_push(y);
set_st(b, b);
for(int x=1; x<=n_x; ++x) g[b][x].w=g[x][b].w=0;
for(int x=1; x<=n; ++x) flo_from[b][x]=0;
for(size_t i=0; i<flo[b].size(); ++i){
    int xs=flo[b][i];
    for(int x=1; x<=n_x; ++x)
        if(g[b][x].w==0 || e_delta(g[xs][x])<e_delta(g[b][x]))
            g[b][x]=g[xs][x], g[x][b]=g[x][xs];
    for(int x=1; x<=n; ++x)
        if(flo_from[xs][x]) flo_from[b][x]=xs;
}
set_slack(b);
}
void expand_blossom(int b){
for(size_t i=0; i<flo[b].size(); ++i)
    set_st(flo[b][i], flo[b][i]);
int xr=flo_from[b][g[b][pa[b]].u], pr=get_pr(b, xr);
for(int i=0; i<pr; i+=2){
    int xs=flo[b][i], xns=flo[b][i+1];
    pa[xs]=g[xns][xs].u;
    S[xs]=1, S[xns]=0;

```

```

    slack[xs]=0, set_slack(xns);
    q_push(xns);
}
S[xr]=1, pa[xr]=pa[b];
for(size_t i=pr+1; i<flo[b].size(); ++i){
    int xs=flo[b][i];
    S[xs]=-1, set_slack(xs);
}
st[b]=0;
}
bool on_found_edge(const edge &e){
int u=st[e.u], v=st[e.v];
if(S[v]==-1){
    pa[v]=e.u, S[v]=1;
    int nu=st[match[v]];
    slack[v]=slack[nu]=0;
    S[nu]=0, q_push(nu);
}else if(S[v]==0){
    int lca=get_lca(u, v);
    if(!lca) return augment(u, v), augment(v, u), true;
    else add_blossom(u, lca, v);
}
return false;
}
bool matching(){
memset(S+1, -1, sizeof(int)*n_x);
memset(slack+1, 0, sizeof(int)*n_x);
q=queue<int>();
for(int x=1; x<=n_x; ++x)
    if(st[x]==x&&!match[x]) pa[x]=0, S[x]=0, q_push(x);
if(q.empty()) return false;
for(;;){
    while(q.size()){
        int u=q.front(); q.pop();
        if(S[st[u]]==1) continue;
        for(int v=1; v<=n; ++v)
            if(g[u][v].w>0&&st[u]!=st[v]){
                if(e_delta(g[u][v])==0){
                    if(on_found_edge(g[u][v])) return true;
                }else update_slack(u, st[v]);
            }
    }
    int d=INF;
    for(int b=n+1; b<=n_x; ++b)
        if(st[b]==b&&S[b]==1) d=min(d, lab[b]/2);
    for(int x=1; x<=n_x; ++x)
        if(st[x]==x&&slack[x]){
            if(S[x]==-1) d=min(d, e_delta(g[slack[x]][x]));
            else if(S[x]==0) d=min(d, e_delta(g[slack[x]][x])/2);
        }
    for(int u=1; u<=n; ++u){
        if(S[st[u]]==0){
            if(lab[u]<=d) return 0;
            lab[u]-=d;
        }else if(S[st[u]]==1) lab[u]+=d;
    }
}

```

```

    for(int b=n+1;b<=n_x;++b)
    if(st[b]==b){
        if(S[st[b]]==0)lab[b]+=d*2;
        else if(S[st[b]]==1)lab[b]-=d*2;
    }
    q=queue<int>();
    for(int x=1;x<=n_x;++x)
    if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta(g[slack[x]] [x])==0)
    if(on_found_edge(g[slack[x]] [x]))return true;
    for(int b=n+1;b<=n_x;++b)
    if(st[b]==b&&S[b]==1&&lab[b]==0)expand_blossom(b);
}
return false;
}
pair<long long,int> solve(){
memset(match+1,0,sizeof(int)*n);
n_x=n;
int n_matches=0;
long long tot_weight=0;
for(int u=0;u<=n;++u)st[u]=u,flo[u].clear();
int w_max=0;
for(int u=1;u<=n;++u)
    for(int v=1;v<=n;++v){
        flo_from[u][v]=(u==v?u:0);
        w_max=max(w_max,g[u][v].w);
    }
for(int u=1;u<=n;++u)lab[u]=w_max;
while(matching())++n_matches;
for(int u=1;u<=n;++u)
    if(match[u]&&match[u]<u)
        tot_weight+=g[u][match[u]].w;
return make_pair(tot_weight,n_matches);
}
void add_edge( int ui , int vi , int wi ){
g[ui][vi].w = g[vi][ui].w = wi;
}
void init( int _n ){
n = _n;
for(int u=1;u<=n;++u)
    for(int v=1;v<=n;++v)
        g[u][v]=edge(u,v,0);
}
} graph;

```

3 Graph

3.1 2-SAT

Should be **revised**.

```

strongly_connected scc;
int n; // = number of clauses
void init(int _n){ scc.clear(); n = _n; }
int NOT(int x){ return x >= n ? (x - n) : (x + n); }
void add_edge(int x, int y){ // input ~x to denote NOT
    if((x >> 31) & 1) x = (~x) + n;

```

```

    if((y >> 31) & 1) y = (~y) + n;
    scc.add_edge(x, y), scc.add_edge(NOT(y), NOT(x));
}
bool satisfy(vector<bool> &res){
    res.resize(n);
    scc.get_scc(2*n);
    for(int i=0; i<n; i++){
        if(scc.comp[i] == scc.comp[NOT(i)]) return 0;
        if(scc.comp[i] < scc.comp[NOT(i)]) res[i] = 0;
        else res[i] = 1;
    }
    return 1;
}

```

3.2 BCC

```

void color(int x, int p){
    if(p){
        bcc[p].push_back(x);
        cmp[x].push_back(p);
    }
    for(auto &i : gph[x]){
        if(cmp[i].size()) continue;
        if(low[i] >= dfn[x]){
            bcc[++c].push_back(x);
            cmp[x].push_back(c);
            color(i, c);
        }
        else color(i, p);
    }
}

```

3.3 Splay Tree + Link-Cut Tree

```

// Checklist 1. Is it link cut, or splay?
// Checklist 2. In link cut, is son always root?
void rotate(node *x){
    if(!x->p) return;
    push(x->p); // if there's lazy stuff
    push(x);
    node *p = x->p;
    bool is_left = (p->l == x);
    node *b = (is_left ? x->r : x->l);
    x->p = p->p;
    if(x->p && x->p->l == p) x->p->l = x;
    if(x->p && x->p->r == p) x->p->r = x;
    if(is_left){
        if(b) b->p = p;
        p->l = b;
        p->p = x;
        x->r = p;
    }
    else{
        if(b) b->p = p;
        p->r = b;
        p->p = x;
        x->l = p;
    }
}

```



```

    x->l = p;
}
pull(p); // if there's something to pull up
pull(x);
if(!x->p) root = x; // IF YOU ARE SPLAY TREE
if(p->pp){ // IF YOU ARE LINK CUT TREE
    x->pp = p->pp;
    p->pp = NULL;
}
}
}
void splay(node *x){
    while(x->p){
        node *p = x->p;
        node *g = p->p;
        if(g){
            if((p->l == x) ^ (g->l == p)) rotate(x);
            else rotate(p);
        }
        rotate(x);
    }
}
void access(node *x){
    splay(x);
    push(x);
    if(x->r){
        x->r->pp = x;
        x->r->p = NULL;
        x->r = NULL;
    }
    pull(x);
    while(x->pp){
        node *nxt = x->pp;
        splay(nxt);
        push(nxt);
        if(nxt->r){
            nxt->r->pp = nxt;
            nxt->r->p = NULL;
            nxt->r = NULL;
        }
        nxt->r = x;
        x->p = nxt;
        x->pp = NULL;
        pull(nxt);
        splay(x);
    }
}
node *root(node *x){
    access(x);
    while(x->l){
        push(x);
        x = x->l;
    }
    access(x);
    return x;
}
node *par(node *x){

```

```

    access(x);
    if(!x->l) return NULL;
    push(x);
    x = x->l;
    while(x->r){
        push(x);
        x = x->r;
    }
    access(x);
    return x;
}
node *lca(node *s, node *t){
    access(s);
    access(t);
    splay(s);
    if(s->pp == NULL) return s;
    return s->pp;
}
void link(node *par, node *son){
    access(par);
    access(son);
    son->rev ^= 1; // remove if needed
    push(son);
    son->l = par;
    par->p = son;
    pull(son);
}
void cut(node *p){
    access(p);
    push(p);
    if(p->l){
        p->l->p = NULL;
        p->l = NULL;
    }
    pull(p);
}

```

3.4 Offline Dynamic MST

Should be **revised**.

```

int n, m, q;
int st[MAXN], ed[MAXN], cost[MAXN], chk[MAXN];
pi qr[MAXN];

bool cmp(int &a, int &b){ return pi(cost[a], a) < pi(cost[b], b); }

void contract(int s, int e, vector<int> v, vector<int> &must_mst, vector<int> &maybe_mst){
    sort(v.begin(), v.end(), cmp);
    vector<pi> snapshot;
    for(int i=s; i<=e; i++) disj.uni(st[qr[i].first], ed[qr[i].first], snapshot);
    for(auto &i : v) if(disj.uni(st[i], ed[i], snapshot)) must_mst.push_back(i);
    disj.revert(snapshot);
    for(auto &i : must_mst) disj.uni(st[i], ed[i], snapshot);
    for(auto &i : v) if(disj.uni(st[i], ed[i], snapshot)) maybe_mst.push_back(i);
    disj.revert(snapshot);
}

```

```

}

void solve(int s, int e, vector<int> v, lint cv){
    if(s == e){
        cost[qr[s].first] = qr[s].second;
        if(st[qr[s].first] == ed[qr[s].first]){
            printf("%lld\n", cv);
            return;
        }
        int minv = qr[s].second;
        for(auto &i : v) minv = min(minv, cost[i]);
        printf("%lld\n", minv + cv);
        return;
    }
    int m = (s+e)/2;
    vector<int> lv = v, rv = v;
    vector<int> must_mst, maybe_mst;
    for(int i=m+1; i<=e; i++){
        chk[qr[i].first]--;
        if(chk[qr[i].first] == 0) lv.push_back(qr[i].first);
    }
    vector<pi> snapshot;
    contract(s, m, lv, must_mst, maybe_mst);
    lint lcv = cv;
    for(auto &i : must_mst) lcv += cost[i], disj.uni(st[i], ed[i], snapshot);
    solve(s, m, maybe_mst, lcv);
    disj.revert(snapshot);
    must_mst.clear(); maybe_mst.clear();
    for(int i=m+1; i<=e; i++) chk[qr[i].first]++;
    for(int i=s; i<=m; i++){
        chk[qr[i].first]--;
        if(chk[qr[i].first] == 0) rv.push_back(qr[i].first);
    }
    lint rcv = cv;
    contract(m+1, e, rv, must_mst, maybe_mst);
    for(auto &i : must_mst) rcv += cost[i], disj.uni(st[i], ed[i], snapshot);
    solve(m+1, e, maybe_mst, rcv);
    disj.revert(snapshot);
    for(int i=s; i<=m; i++) chk[qr[i].first]++;
}

int main(){
    scanf("%d %d", &n, &m);
    vector<int> ve;
    for(int i=0; i<m; i++){
        scanf("%d %d %d", &st[i], &ed[i], &cost[i]);
    }
    scanf("%d", &q);
    for(int i=0; i<q; i++){
        scanf("%d %d", &qr[i].first, &qr[i].second);
        qr[i].first--;
        chk[qr[i].first]++;
    }
    disj.init(n);
    for(int i=0; i<m; i++) if(!chk[i]) ve.push_back(i);
    solve(0, q-1, ve, 0);
}

```

```

}

```

3.5 Dominator Tree

```

// CREDITS: cki86201
vector<int> E[MAXN], RE[MAXN], rdom[MAXN];

int S[MAXN], RS[MAXN], cs;
int par[MAXN], val[MAXN], sdom[MAXN], rp[MAXN], dom[MAXN];

void clear(int n) {
    cs = 0;
    for(int i=0; i<=n; i++) {
        par[i] = val[i] = sdom[i] = rp[i] = dom[i] = S[i] = RS[i] = 0;
        E[i].clear(); RE[i].clear(); rdom[i].clear();
    }
}

void add_edge(int x, int y) { E[x].push_back(y); }
void Union(int x, int y) { par[x] = y; }
int Find(int x, int c = 0) {
    if(par[x] == x) return c ? -1 : x;
    int p = Find(par[x], 1);
    if(p == -1) return c ? par[x] : val[x];
    if(sdom[val[x]] > sdom[val[par[x]]]) val[x] = val[par[x]];
    par[x] = p;
    return c ? p : val[x];
}

void dfs(int x) {
    RS[S[x] = ++cs] = x;
    par[cs] = sdom[cs] = val[cs] = cs;
    for(int e : E[x]) {
        if(S[e] == 0) dfs(e), rp[S[e]] = S[x];
        RE[S[e]].push_back(S[x]);
    }
}

int solve(int s, int *up) { // Calculate idoms
    dfs(s);
    for(int i=cs; i-->0) {
        for(int e : RE[i]) sdom[i] = min(sdom[i], sdom[Find(e)]);
        if(i > 1) rdom[sdom[i]].push_back(i);
        for(int e : rdom[i]) {
            int p = Find(e);
            if(sdom[p] == i) dom[e] = i;
            else dom[e] = p;
        }
        if(i > 1) Union(i, rp[i]);
    }
    for(int i=2; i<=cs; i++) if(sdom[i] != dom[i]) dom[i] = dom[dom[i]];
    for(int i=2; i<=cs; i++) up[RS[i]] = RS[dom[i]];
    return cs;
}

```

3.6 Global Min-Cut

```

int minimum_cut_phase(int n, int &s, int &t, vector<vector<int>> &adj, vector<int> vis){
    vector<int> dist(n);
}

```

```

int mincut = 1e9;
while(true){
    int pos = -1, cur = -1e9;
    for(int i=0; i<n; i++){
        if(!vis[i] && dist[i] > cur){
            cur = dist[i];
            pos = i;
        }
    }
    if(pos == -1) break;
    s = t;
    t = pos;
    mincut = cur;
    vis[pos] = 1;
    for(int i=0; i<n; i++){
        if(!vis[i]) dist[i] += adj[pos][i];
    }
}
return mincut; // optimal s-t cut here is, {t} and V \ {t}
}

int solve(int n, vector<vector<int>> adj){
    if(n <= 1) return 0;
    vector<int> vis(n);
    int ans = 1e9;
    for(int i=0; i<n-1; i++){
        int s, t;
        ans = min(ans, minimum_cut_phase(n, s, t, adj, vis));
        vis[t] = 1;
        for(int j=0; j<n; j++){
            if(!vis[j]){
                adj[s][j] += adj[t][j];
                adj[j][s] += adj[j][t];
            }
        }
        adj[s][s] = 0;
    }
    return ans;
}

```

3.7 Edmond's Directed MST

```

// starts from node 0. assumes there exists at least one dmst.
// edge is reversed : if there is edge s -> e, INSERT IN gph[e]
struct edge { int to, cost, id; };
using elist = vector<edge>;
void dmst(vector<elist> &g, vector<int>& res) {
    const int n = g.size();
    vector<edge*> to(n);
    vector<int> u(n, 0);
    for (int i = 1; i < n; ++i) {
        int mn = g[i][0].cost;
        for (int j = 0; j < g[i].size(); ++j)
            mn = min(mn, g[i][j].cost);
        for (int j = 0; j < g[i].size(); ++j) {
            if (g[i][j].cost == mn)
                to[i] = &g[i][j];
        }
    }
}

```

```

        g[i][j].cost -= mn;
    }
}

for (int i = 1; i < n; ++i) {
    if (u[i]) continue;
    int x = i;
    vector<int> order(1, x);
    u[x] = 1;
    while (to[x]->to > 0 && u[to[x]->to] == 0) {
        x = to[x]->to;
        u[x] = 1;
        order.push_back(x);
    }
    int y = to[x]->to;
    vector<int> cycle(find(order.begin(), order.end(), y), order.end());
    if (cycle.size() == 0) continue;
    vector<int> in_cycle(n);
    for (int j = 0; j < cycle.size(); ++j) in_cycle[cycle[j]] = 1;
    vector<int> nw_id(n);
    for (int j = 1; j < n; ++j) nw_id[j] = nw_id[j - 1] + !in_cycle[j];
    int nn = n - cycle.size();
    vector<elist> gn(nn + 1);
    for (int j = 1; j < n; ++j) {
        if (in_cycle[j]) {
            for (int k = 0; k < g[j].size(); ++k)
                if (!in_cycle[g[j][k].to])
                    gn[nn].push_back({nw_id[g[j][k].to], g[j][k].cost, g[j][k].id});
        } else {
            for (int k = 0; k < g[j].size(); ++k)
                if (in_cycle[g[j][k].to])
                    gn[nw_id[j]].push_back({nn, g[j][k].cost, g[j][k].id});
                else
                    gn[nw_id[j]].push_back({nw_id[g[j][k].to], g[j][k].cost, g[j][k].id});
        }
    }
}

dmst(gn, res);
set<int> used_e(res.begin(), res.end());
for (int j = 0; j < cycle.size(); ++j) {
    bool found = false;
    for (int k = 0; k < g[cycle[j]].size(); ++k)
        found |= used_e.count(g[cycle[j]][k].id);
    if (found) {
        for (int k = 0; k < cycle.size(); ++k) {
            if (k != j)
                res.push_back(to[cycle[k]]->id);
        }
        return;
    }
}
}

for (int i = 1; i < n; ++i) res.push_back(to[i]->id);
}

```

4 Strings

4.1 Aho-Corasick Algorithm

```
const int MAXN = 100005, MAXC = 26;
int trie[MAXN][MAXC], fail[MAXN], term[MAXN], piv;
void init(vector<string> &v){
    memset(trie, 0, sizeof(trie));
    memset(fail, 0, sizeof(fail));
    memset(term, 0, sizeof(term));
    piv = 0;
    for(auto &i : v){
        int p = 0;
        for(auto &j : i){
            if(!trie[p][j]) trie[p][j] = ++piv;
            p = trie[p][j];
        }
        term[p] = 1;
    }
    queue<int> que;
    for(int i=0; i<MAXC; i++){
        if(trie[0][i]) que.push(trie[0][i]);
    }
    while(!que.empty()){
        int x = que.front();
        que.pop();
        for(int i=0; i<MAXC; i++){
            if(trie[x][i]){
                int p = fail[x];
                while(p && !trie[p][i]) p = fail[p];
                p = trie[p][i];
                fail[trie[x][i]] = p;
                if(term[p]) term[trie[x][i]] = 1;
                que.push(trie[x][i]);
            }
        }
    }
}
bool query(string &s){
    int p = 0;
    for(auto &i : s){
        while(p && !trie[p][i]) p = fail[p];
        p = trie[p][i];
        if(term[p]) return 1;
    }
    return 0;
}
```

4.2 Suffix Array

Should be **revised**.

```
const int MAXN = 500005;
int ord[MAXN], nord[MAXN], cnt[MAXN], aux[MAXN];
void solve(int n, char *str, int *sfx, int *rev, int *lcp){
    int p = 1;
    memset(ord, 0, sizeof(ord));
```

```
for(int i=0; i<n; i++){
    sfx[i] = i;
    ord[i] = str[i];
}
int pnt = 1;
while(1){
    memset(cnt, 0, sizeof(cnt));
    for(int i=0; i<n; i++) cnt[ord[min(i+p, n)]]++;
    for(int i=1; i<=n || i<=255; i++) cnt[i] += cnt[i-1];
    for(int i=n-1; i>=0; i--){
        aux[--cnt[ord[min(i+p, n)]]] = i;
    }
    memset(cnt, 0, sizeof(cnt));
    for(int i=0; i<n; i++) cnt[ord[i]]++;
    for(int i=1; i<=n || i<=255; i++) cnt[i] += cnt[i-1];
    for(int i=n-1; i>=0; i--){
        sfx[--cnt[ord[aux[i]]]] = aux[i];
    }
    if(pnt == n) break;
    pnt = 1;
    nord[sfx[0]] = 1;
    for(int i=1; i<n; i++){
        if(ord[sfx[i-1]] != ord[sfx[i]] || ord[sfx[i-1] + p] != ord[sfx[i] + p]){
            pnt++;
        }
        nord[sfx[i]] = pnt;
    }
    memcpy(ord, nord, sizeof(int) * n);
    p *= 2;
}
for(int i=0; i<n; i++) rev[sfx[i]] = i;
int h = 0;
for(int i=0; i<n; i++){
    if(rev[i]){
        int prv = sfx[rev[i] - 1];
        while(str[prv + h] == str[i + h]) h++;
        lcp[rev[i]] = h;
    }
    h = max(h-1, 0);
}
}
```

4.3 Manacher's Algorithm

```
const int MAXN = 1000005;
int aux[2 * MAXN - 1];
void solve(int n, int *str, int *ret){
    // *ret : number of nonobvious palindromic character pair
    for(int i=0; i<n; i++){
        aux[2*i] = str[i];
        if(i != n-1) aux[2*i+1] = -1;
    }
    int p = 0, c = 0;
    for(int i=0; i<2*n-1; i++){
        int cur = 0;
        if(i <= p) cur = min(ret[2 * c - i], p - i);
        while(i - cur - 1 >= 0 && i + cur + 1 < 2*n-1 && aux[i-cur-1] == aux[i+cur+1]){
            cur++;
        }
    }
}
```

```

    }
    ret[i] = cur;
    if(i + ret[i] > p){
        p = i + ret[i];
        c = i;
    }
}
}

```

4.4 eertree

```

int nxt[MAXN][26];
int par[MAXN], len[MAXN], slink[MAXN], ptr[MAXN], diff[MAXN], series[MAXN], piv;
void clear(int n = MAXN){
    memset(par, 0, sizeof(int) * n);
    memset(len, 0, sizeof(int) * n);
    memset(slink, 0, sizeof(int) * n);
    memset(nxt, 0, sizeof(int) * 26 * n);
    piv = 0;
}
void init(int n, char *a){
    par[0] = 0;
    par[1] = 1;
    a[0] = -1;
    len[0] = -1;
    piv = 1;
    int cur = 1;
    for(int i=1; i<=n; i++){
        while(a[i] != a[i - len[cur] - 1]) cur = slink[cur];
        if(!nxt[cur][a[i]]){
            nxt[cur][a[i]] = ++piv;
            par[piv] = cur;
            len[piv] = len[cur] + 2;
            int lnk = slink[cur];
            while(a[i] != a[i - len[lnk] - 1]){
                lnk = slink[lnk];
            }
            if(nxt[lnk][a[i]]) lnk = nxt[lnk][a[i]];
            if(len[piv] == 1 || lnk == 0) lnk = 1;
            slink[piv] = lnk;
            diff[piv] = len[piv] - len[lnk];
            if(diff[piv] == diff[lnk]) series[piv] = series[lnk];
            else series[piv] = piv;
        }
        cur = nxt[cur][a[i]];
        ptr[i] = cur;
    }
}
int query(int s, int e){
    int pos = ptr[e];
    while(len[pos] >= e - s + 1){
        if(len[pos] % diff[pos] == (e - s + 1) % diff[pos] &&
            len[series[pos]] <= e - s + 1) return true;
        pos = series[pos];
        pos = slink[pos];
    }
}

```

```

    return false;
}
vector<pi> minimum_partition(int n){ // (odd min, even min)
    vector<pi> dp(n + 1);
    vector<pi> series_ans(n + 10);
    dp[0] = pi(1e9 + 1, 0);
    for(int i=1; i<=n; i++){
        dp[i] = pi(1e9 + 1, 1e9);
        for(int j=ptr[i]; len[j] > 0;){
            int slv = slink[series[j]];
            series_ans[j] = dp[i - (len[slv] + diff[j])];
            if(diff[j] == diff[slink[j]]){
                series_ans[j].first = min(series_ans[j].first, series_ans[slink[j]].first);
                series_ans[j].second = min(series_ans[j].second, series_ans[slink[j]].second);
            }
            auto val = series_ans[j];
            dp[i].first = min(dp[i].first, val.second + 1);
            dp[i].second = min(dp[i].second, val.first + 1);
            j = slv;
        }
    }
    return dp;
}

```

4.5 Circular LCS

```
string s1, s2;
```

```

int dp[4005][2005];
int nxt[4005][2005];
int n, m;

void reroot(int px){
    int py = 1;
    while(py <= m && nxt[px][py] != 2) py++;
    nxt[px][py] = 1;
    while(px < 2 * n && py < m){
        if(nxt[px+1][py] == 3){
            px++;
            nxt[px][py] = 1;
        }
        else if(nxt[px+1][py+1] == 2){
            px++;
            py++;
            nxt[px][py] = 1;
        }
        else py++;
    }
    while(px < 2 * n && nxt[px+1][py] == 3){
        px++;
        nxt[px][py] = 1;
    }
}

int track(int x, int y, int e){ // use this routine to find LCS as string
    int ret = 0;
}

```

```

while(y != 0 && x != e){
    if(nxt[x][y] == 1) y--;
    else if(nxt[x][y] == 2) ret += (s1[x] == s2[y]), x--, y--;
    else if(nxt[x][y] == 3) x--;
}
return ret;
}

int solve(string a, string b){
    n = a.size(), m = b.size();
    s1 = "#" + a + a;
    s1 = '#' + b;
    for(int i=0; i<=2*n; i++){
        for(int j=0; j<=m; j++){
            if(j == 0){
                nxt[i][j] = 3;
                continue;
            }
            if(i == 0){
                nxt[i][j] = 1;
                continue;
            }
            dp[i][j] = -1;
            if(dp[i][j] < dp[i][j-1]){
                dp[i][j] = dp[i][j-1];
                nxt[i][j] = 1;
            }
            if(dp[i][j] < dp[i-1][j-1] + (s1[i] == s2[j])){
                dp[i][j] = dp[i-1][j-1] + (s1[i] == s2[j]);
                nxt[i][j] = 2;
            }
            if(dp[i][j] < dp[i-1][j]){
                dp[i][j] = dp[i-1][j];
                nxt[i][j] = 3;
            }
        }
    }
    int ret = dp[n][m];
    for(int i=1; i<n; i++){
        reroot(i), ret = max(ret, track(n+i, m, i));
    }
    return ret;
}

```

5 Geometry

5.1 Green's Theorem

Let C is positive, smooth, simple curve. D is region bounded by C .

$$\oint_C (Ldx + Mdy) = \iint_D \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \right)$$

To calculate area, $\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} = 1$, common selection is $M = \frac{1}{2}x$, $L = -\frac{1}{2}y$.

Line integral of circle parametrized by $(x, y) = (x_C + r_C \cos \theta, y_C + r_C \sin \theta)$ is given as follows.:

$$\frac{1}{2}(r_C(x_C(\sin \theta_f - \sin \theta_i) - y_C(\cos \theta_f - \cos \theta_i)) + (\theta_f - \theta_i)r_C^2).$$

Line integral of line parametrized by $(x, y) = t(x_1, y_1) + (1-t)(x_2, y_2)$ is given as follows.: $\frac{1}{2}(x_1y_2 - x_2y_1)$.

```

inline double arc_area(double x, double y, double r, double s, double e){ // s and e are
line integral theta value.
    return (r*(x*(sin(e)-sin(s))-y*(cos(e)-cos(s)))+(e-s)*r*r)*0.5;
}

inline double polygon_area(double x1, double y1, double x2, double y2){ // Shoelace formula
    return (x1*y2-x2*y1)*0.5;
}

```

5.2 Smallest Enclosing Circle / Sphere

```

namespace cover_2d{
    double eps = 1e-9;
    using Point = complex<double>;
    struct Circle{ Point p; double r; };
    double dist(Point p, Point q){ return abs(p-q); }
    double area2(Point p, Point q){ return (conj(p)*q).imag(); }
    bool in(const Circle& c, Point p){ return dist(c.p, p) < c.r + eps; }
    Circle INVALID = Circle{Point(0, 0), -1};
    Circle mCC(Point a, Point b, Point c){
        b -= a; c -= a;
        double d = 2*(conj(b)*c).imag(); if(abs(d)<eps) return INVALID;
        Point ans = (c*norm(b) - b*norm(c)) * Point(0, -1) / d;
        return Circle{a + ans, abs(ans)};
    }
    Circle solve(vector<Point> p) {
        mt19937 gen(0x94949); shuffle(p.begin(), p.end(), gen);
        Circle c = INVALID;
        for(int i=0; i<p.size(); ++i) if(c.r<0 || !in(c, p[i])){
            c = Circle{p[i], 0};
            for(int j=0; j<=i; ++j) if(!in(c, p[j])){
                Circle ans{(p[i]+p[j])*0.5, dist(p[i], p[j])*0.5};
                if(c.r == 0) { c = ans; continue; }
                Circle l, r; l = r = INVALID;
                Point pq = p[j]-p[i];
                for(int k=0; k<=j; ++k) if(!in(ans, p[k])) {
                    double a2 = area2(pq, p[k]-p[i]);
                    Circle c = mCC(p[i], p[j], p[k]);
                    if(c.r<0) continue;
                    else if(a2 > 0 && (l.r<0 || area2(pq, c.p-p[i]) > area2(pq, l.p-p[i]))) l = c;
                    else if(a2 < 0 && (r.r<0 || area2(pq, c.p-p[i]) < area2(pq, r.p-p[i]))) r = c;
                }
                if(l.r<0&&r.r<0) c = ans;
                else if(l.r<0) c = r;
                else if(r.r<0) c = l;
                else c = l.r<=r.r?l:r;
            }
        }
        return c;
    }
}

```

```

namespace cover_3d{
    double enclosing_sphere(vector<double> x, vector<double> y, vector<double> z){
        int n = x.size();
        auto hyp = [](double x, double y, double z){
            return x * x + y * y + z * z;
        };
        double px = 0, py = 0, pz = 0;
        for(int i=0; i<n; i++){
            px += x[i];
            py += y[i];
            pz += z[i];
        }
        px /= n;
        py /= n;
        pz /= n;
        double rat = 0.1, maxv;
        for(int i=0; i<10000; i++){
            maxv = -1;
            int maxp = -1;
            for(int j=0; j<n; j++){
                double tmp = hyp(x[j] - px, y[j] - py, z[j] - pz);
                if(maxv < tmp){
                    maxv = tmp;
                    maxp = j;
                }
            }
            px += (x[maxp] - px) * rat;
            py += (y[maxp] - py) * rat;
            pz += (z[maxp] - pz) * rat;
            rat *= 0.998;
        }
        return sqrt(maxv);
    }
};

```

5.3 3D Convex Hull

// CREDITS: <https://gist.github.com/msg555/4963794>

```

struct vec3{
    ll x, y, z;
    vec3(): x(0), y(0), z(0) {}
    vec3(ll a, ll b, ll c): x(a), y(b), z(c) {}
    vec3 operator*(const vec3& v) const{ return vec3(y*v.z-z*v.y, z*v.x-x*v.z, x*v.y-y*v.x); }
    vec3 operator-(const vec3& v) const{ return vec3(x-v.x, y-v.y, z-v.z); }
    vec3 operator-() const{ return vec3(-x, -y, -z); }
    ll dot(const vec3 &v) const{ return x*v.x+y*v.y+z*v.z; }
};

```

```

struct twoset {
    int a, b;
    void insert(int x) { (a == -1 ? a : b) = x; }
    bool contains(int x) { return a == x || b == x; }
    void erase(int x) { (a == x ? a : b) = -1; }
    int size() { return (a != -1) + (b != -1); }
} E[MAXN][MAXN]; // i < j

```

```

struct face{
    vec3 norm;
    ll disc;
    int I[3];
};

face make_face(int i, int j, int k, int ii, vector<vec3> &A){ // p^T * norm < disc
    E[i][j].insert(k); E[i][k].insert(j); E[j][k].insert(i);
    face f; f.I[0]=i, f.I[1]=j, f.I[2]=k;
    f.norm = (A[j]-A[i])*(A[k]-A[i]);
    f.disc = f.norm.dot(A[i]);
    if(f.norm.dot(A[ii])>f.disc){
        f.norm = -f.norm;
        f.disc = -f.disc;
    }
    return f;
}

```

```

vector<face> get_hull(vector<vec3> &A){
    int N = A.size();
    vector<face> faces; memset(E, -1, sizeof(E));
    faces.push_back(make_face(0,1,2,3,A));
    faces.push_back(make_face(0,1,3,2,A));
    faces.push_back(make_face(0,2,3,1,A));
    faces.push_back(make_face(1,2,3,0,A));
    for(int i=4; i<N; ++i){
        for(int j=0; j<faces.size(); ++j){
            face f = faces[j];
            if(f.norm.dot(A[i])>f.disc){
                E[f.I[0]][f.I[1]].erase(f.I[2]);
                E[f.I[0]][f.I[2]].erase(f.I[1]);
                E[f.I[1]][f.I[2]].erase(f.I[0]);
                faces[j--] = faces.back();
                faces.pop_back();
            }
        }
        int nf = faces.size();
        for(int j=0; j<nf; ++j){
            face f=faces[j];
            for(int a=0; a<3; ++a) for(int b=a+1; b<3; ++b){
                int c=3-a-b;
                if(E[f.I[a]][f.I[b]].size()==2) continue;
                faces.push_back(make_face(f.I[a], f.I[b], i, f.I[c], A));
            }
        }
        return faces;
    }
}

```

5.4 Dynamic Convex Hull Trick

// CREDITS: https://github.com/niklasb/contest-algos/blob/master/convex_hull/dynamic.cpp
 using line_t = double;
 const line_t is_query = -1e18;

```

struct Line {
    line_t m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        if (!s) return 0;
        line_t x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};

struct HullDynamic : public multiset<Line> { // will maintain upper hull for maximum
    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if (z == end()) return y->m == x->m && y->b <= x->b;
        return (x->b - y->b)*(z->m - y->m) >= (y->b - z->b)*(y->m - x->m);
    }
    void insert_line(line_t m, line_t b) {
        auto y = insert({ m, b });
        y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
        if (bad(y)) { erase(y); return; }
        while (next(y) != end() && bad(next(y))) erase(next(y));
        while (y != begin() && bad(prev(y))) erase(prev(y));
    }
    line_t query(line_t x) {
        auto l = *lower_bound((Line) { x, is_query });
        return l.m * x + l.b;
    }
};

```

5.5 Half-plane Intersection

```

const double eps = 1e-8;
typedef pair<long double, long double> pi;
bool z(long double x){ return fabs(x) < eps; }
struct line{
    long double a, b, c;
    bool operator<(const line &l) const{
        bool flag1 = pi(a, b) > pi(0, 0);
        bool flag2 = pi(l.a, l.b) > pi(0, 0);
        if(flag1 != flag2) return flag1 > flag2;
        long double t = ccw(pi(0, 0), pi(a, b), pi(l.a, l.b));
        return z(t) ? c * hypot(l.a, l.b) < l.c * hypot(a, b) : t > 0;
    }
    pi slope(){ return pi(a, b); }
};
pi cross(line a, line b){
    long double det = a.a * b.b - b.a * a.b;
    return pi((a.c * b.b - a.b * b.c) / det, (a.a * b.c - a.c * b.a) / det);
}

```

```

bool bad(line a, line b, line c){
    if(ccw(pi(0, 0), a.slope(), b.slope()) <= 0) return false;
    pi crs = cross(a, b);
    return crs.first * c.a + crs.second * c.b >= c.c;
}

bool solve(vector<line> v, vector<pi> &solution){ // ax + by <= c;
    sort(v.begin(), v.end());
    deque<line> dq;
    for(auto &i : v){
        if(!dq.empty() && z(ccw(pi(0, 0), dq.back().slope(), i.slope())) continue;
        while(dq.size() >= 2 && bad(dq[dq.size()-2], dq.back(), i)) dq.pop_back();
        while(dq.size() >= 2 && bad(i, dq[0], dq[1])) dq.pop_front();
        dq.push_back(i);
    }
    while(dq.size() > 2 && bad(dq[dq.size()-2], dq.back(), dq[0])) dq.pop_back();
    while(dq.size() > 2 && bad(dq.back(), dq[0], dq[1])) dq.pop_front();
    vector<pi> tmp;
    for(int i=0; i<dq.size(); i++){
        line cur = dq[i], nxt = dq[(i+1)%dq.size()];
        if(ccw(pi(0, 0), cur.slope(), nxt.slope()) <= eps) return false;
        tmp.push_back(cross(cur, nxt));
    }
    solution = tmp;
    return true;
}

```

5.6 Point-in-polygon test / Point-to-polygon tangent

```

// CREDITS: zigui
// C : counter_clockwise(C[0] == C[N]), N >= 3
// return highest point in C <- P(clockwise) or -1 if strictly in P
// polygon is strongly convex, C[i] != P

```

```

int convex_tangent(vector<pi> &C, pi P, int up = 1){
    auto sign = [&](int c){ return c > 0 ? up : c == 0 ? 0 : -up; };
    auto local = [&](pi P, pi a, pi b, pi c) {
        return sign(ccw(P, a, b)) <= 0 && sign(ccw(P, b, c)) >= 0;
    };
    int N = C.size()-1, s = 0, e = N, m;
    if( local(P, C[1], C[0], C[N-1]) ) return 0;
    while(s+1 < e){
        m = (s+e) / 2;
        if( local(P, C[m-1], C[m], C[m+1]) ) return m;
        if( sign(ccw(P, C[s], C[s+1])) < 0 ){ // up
            if( sign(ccw(P, C[m], C[m+1])) > 0 ) e = m;
            else if( sign(ccw(P, C[m], C[s])) > 0 ) s = m;
            else e = m;
        }
        else{ // down
            if( sign(ccw(P, C[m], C[m+1])) < 0 ) s = m;
            else if( sign(ccw(P, C[m], C[s])) < 0 ) s = m;
            else e = m;
        }
    }
    if( s && local(P, C[s-1], C[s], C[s+1]) ) return s;
    if( e != N && local(P, C[e-1], C[e], C[e+1]) ) return e;
}

```



```
    return -1;
}
```

5.7 kd-tree

```
typedef pair<int, int> pi;
struct node{
    pi pnt;
    int spl, sx, ex, sy, ey;
}tree[270000];

pi a[100005];
int n, ok[270000];

lint sqr(int x){ return 1ll * x * x; }
bool cmp1(pi a, pi b){ return a < b; }
bool cmp2(pi a, pi b){ return pi(a.second, a.first) < pi(b.second, b.first); }

// init(0, n-1, 1) : Initialize kd-tree
// set dap = INF, and call solve(1, P). dap = (closest point from P)

void init(int s, int e, int p){ // Initialize kd-tree
    int minx = 1e9, maxx = -1e9, miny = 1e9, maxy = -1e9;
    int m = (s+e)/2;
    for(int i=s; i<=e; i++){
        minx = min(minx, a[i].first);
        miny = min(miny, a[i].second);
        maxx = max(maxx, a[i].first);
        maxy = max(maxy, a[i].second);
    }
    tree[p].spl = (maxx - minx < maxy - miny);
    sort(a+s, a+e+1, [&](const pi &a, const pi &b){
        return tree[p].spl ? cmp2(a, b) : cmp1(a, b);
    });
    ok[p] = 1;
    tree[p] = {a[m], tree[p].spl, minx, maxx, miny, maxy};
    if(s <= m-1) init(s, m-1, 2*p);
    if(m+1 <= e) init(m+1, e, 2*p+1);
}

lint dap = 3e18;

void solve(int p, pi x){ // find closest point from point x (L^2)
    if(x != tree[p].pnt) dap = min(dap, sqr(x.first - tree[p].pnt.first) + sqr(x.second - tree[p].pnt.second));
    if(tree[p].spl){
        if(!cmp2(tree[p].pnt, x)){
            if(ok[2*p]) solve(2*p, x);
            if(ok[2*p+1] && sqr(tree[2*p+1].sy - x.second) < dap) solve(2*p+1, x);
        }
        else{
            if(ok[2*p+1]) solve(2*p+1, x);
            if(ok[2*p] && sqr(tree[2*p].ey - x.second) < dap) solve(2*p, x);
        }
    }
    else{

```

```
        if(!cmp1(tree[p].pnt, x)){
            if(ok[2*p]) solve(2*p, x);
            if(ok[2*p+1] && sqr(tree[2*p+1].sx - x.first) < dap) solve(2*p+1, x);
        }
        else{
            if(ok[2*p+1]) solve(2*p+1, x);
            if(ok[2*p] && sqr(tree[2*p].ex - x.first) < dap) solve(2*p, x);
        }
    }
}
```

6 Math

6.1 FFT / NTT

```
typedef complex<double> base;
void fft(vector<base> &a, bool inv){
    int n = a.size(), j = 0;
    vector<base> roots(n/2);
    for(int i=1; i<n; i++){
        int bit = (n >> 1);
        while(j >= bit){
            j -= bit;
            bit >>= 1;
        }
        j += bit;
        if(i < j) swap(a[i], a[j]);
    }
    double ang = 2 * acos(-1) / n * (inv ? -1 : 1);
    for(int i=0; i<n/2; i++){
        roots[i] = base(cos(ang * i), sin(ang * i));
    }
    /* In NTT, let prr = primitive root. Then,
    int ang = ipow(prr, (mod - 1) / n);
    if(inv) ang = ipow(ang, mod - 2);
    for(int i=0; i<n/2; i++){
        roots[i] = (i ? (1ll * roots[i-1] * ang % mod) : 1);
    }
    Others are same. If there is /= n, do *= ipow(n, mod - 2).
    In XOR convolution, roots[*] = 1.
    */
    for(int i=2; i<=n; i<=1){
        int step = n / i;
        for(int j=0; j<n; j+=i){
            for(int k=0; k<i/2; k++){
                base u = a[j+k], v = a[j+k+i/2] * roots[step * k];
                a[j+k] = u+v;
                a[j+k+i/2] = u-v;
            }
        }
    }
    if(inv) for(int i=0; i<n; i++) a[i] /= n;
}

vector<lint> multiply(vector<lint> &v, vector<lint> &w){
    vector<base> fv(v.begin(), v.end()), fw(w.begin(), w.end());

```

```

    int n = 2; while(n < v.size() + w.size()) n <= 1;
    fv.resize(n); fw.resize(n);
    fft(fv, 0); fft(fw, 0);
    for(int i=0; i<n; i++) fv[i] *= fw[i];
    fft(fv, 1);
    vector<lint> ret(n);
    for(int i=0; i<n; i++) ret[i] = (lint)round(fv[i].real());
    return ret;
}

vector<lint> multiply(vector<lint> &v, vector<lint> &w, lint mod){
    int n = 2; while(n < v.size() + w.size()) n <= 1;
    vector<base> v1(n), v2(n), r1(n), r2(n);
    for(int i=0; i<v.size(); i++){
        v1[i] = base(v[i] >> 15, v[i] & 32767);
    }
    for(int i=0; i<w.size(); i++){
        v2[i] = base(w[i] >> 15, w[i] & 32767);
    }
    fft(v1, 0);
    fft(v2, 0);
    for(int i=0; i<n; i++){
        int j = (i ? (n - i) : i);
        base ans1 = (v1[i] + conj(v1[j])) * base(0.5, 0);
        base ans2 = (v1[i] - conj(v1[j])) * base(0, -0.5);
        base ans3 = (v2[i] + conj(v2[j])) * base(0.5, 0);
        base ans4 = (v2[i] - conj(v2[j])) * base(0, -0.5);
        r1[i] = (ans1 * ans3) + (ans1 * ans4) * base(0, 1);
        r2[i] = (ans2 * ans3) + (ans2 * ans4) * base(0, 1);
    }
    fft(r1, 1);
    fft(r2, 1);
    vector<lint> ret(n);
    for(int i=0; i<n; i++){
        lint av = (lint)round(r1[i].real());
        lint bv = (lint)round(r1[i].imag()) + (lint)round(r2[i].real());
        lint cv = (lint)round(r2[i].imag());
        av %= mod, bv %= mod, cv %= mod;
        ret[i] = (av << 30) + (bv << 15) + cv;
        ret[i] %= mod;
        ret[i] += mod;
        ret[i] %= mod;
    }
    return ret;
}

```

6.2 Hell-Joseon style FFT

```

#include <smmintrin.h>
#include <immintrin.h>
#pragma GCC target("avx2")
#pragma GCC target("fma")
__m256d mult(__m256d a, __m256d b){
    __m256d c = _mm256_movedup_pd(a);
    __m256d d = _mm256_shuffle_pd(a, a, 15);
    __m256d cb = _mm256_mul_pd(c, b);
    __m256d db = _mm256_mul_pd(d, b);

```

```

    __m256d e = _mm256_shuffle_pd(db, db, 5);
    __m256d r = _mm256_addsub_pd(cb, e);
    return r;
}

void fft(int n, __m128d a[], bool invert){
    for(int i=1, j=0; i<n; ++i){
        int bit = n>>1;
        for(;j>=bit;bit>>=1) j -= bit;
        j += bit;
        if(i<j) swap(a[i], a[j]);
    }
    for(int len=2; len<=n; len<=1){
        double ang = 2*3.14159265358979/len*(invert?-1:1);
        __m256d wlen; wlen[0] = cos(ang), wlen[1] = sin(ang);
        for(int i=0; i<n; i += len){
            __m256d w; w[0] = 1; w[1] = 0;
            for(int j=0; j<len/2; ++j){
                w = _mm256_permute2f128_pd(w, w, 0);
                wlen = _mm256_insertf128_pd(wlen, a[i+j+len/2], 1);
                w = mult(w, wlen);
                __m128d vw = _mm256_extractf128_pd(w, 1);
                __m128d u = a[i+j];
                a[i+j] = _mm_add_pd(u, vw);
                a[i+j+len/2] = _mm_sub_pd(u, vw);
            }
        }
    }
    if(invert){
        __m128d inv; inv[0] = inv[1] = 1.0/n;
        for(int i=0; i<n; ++i) a[i] = _mm_mul_pd(a[i], inv);
    }
}

vector<int64_t> multiply(vector<int64_t> &v, vector<int64_t> &w){
    int n = 2; while(n < v.size() + w.size()) n <= 1;
    __m128d* fv = new __m128d[n];
    for(int i=0; i<n; ++i) fv[i][0] = fv[i][1] = 0;
    for(int i=0; i<v.size(); ++i) fv[i][0] = v[i];
    for(int i=0; i<w.size(); ++i) fv[i][1] = w[i];
    fft(n, fv, 0); // (a+bi) is stored in FFT
    for(int i=0; i<n; i += 2){
        __m256d a;
        a = _mm256_insertf128_pd(a, fv[i], 0);
        a = _mm256_insertf128_pd(a, fv[i+1], 1);
        a = mult(a, a);
        fv[i] = _mm256_extractf128_pd(a, 0);
        fv[i+1] = _mm256_extractf128_pd(a, 1);
    }
    fft(n, fv, 1);
    vector<int64_t> ret(n);
    for(int i=0; i<n; ++i) ret[i] = (int64_t)round(fv[i][1]/2);
    delete[] fv;
    return ret;
}

```

6.3 NTT Polynomial Division

```
vector<lint> get_inv(int n, const vector<lint> &p){
    vector<lint> q = {ipow(p[0], mod - 2)};
    for(int i=2; i<=n; i<=1){
        vector<lint> res;
        vector<lint> fq(q.begin(), q.end()); fq.resize(2*i);
        vector<lint> fp(p.begin(), p.begin() + i); fp.resize(2*i);
        fft(fq, 0); fft(fp, 0);
        for(int j=0; j<2*i; j++){
            fp[j] *= fq[j] * fq[j] % mod;
            fp[j] %= mod;
        }
        fft(fp, 1);
        res.resize(i);
        for(int j=0; j<i; j++){
            res[j] = mod - fp[j];
            if(j < i/2) res[j] += 2 * q[j];
            res[j] %= mod;
        }
        q = res;
    }
    return q;
}

vector<lint> poly_divide(const vector<lint> &a, const vector<lint> &b){
    assert(b.back() != 0); // please trim leading zero
    int n = a.size(), m = b.size();
    int k = 2; while(k < n-m+1) k <= 1;
    vector<lint> rb(k), ra(k);
    for(int i=0; i<m && i<k; ++i) rb[i] = b[m-i-1];
    for(int i=0; i<n && i<k; ++i) ra[i] = a[n-i-1];
    vector<lint> rbi = get_inv(k, rb);
    vector<lint> res = multiply(rbi, ra);
    res.resize(n - m + 1);
    reverse(res.begin(), res.end());
    return res;
}
```

6.4 Simplex Algorithm

```
/* Ax <= b, max c^T x
 * Usage : Simplex(VVD A, VD b, VD c).solve(VD ans)
 * not feasible : -INF; unbounded : INF
 * accuracy ~ (size of ans) * EPS
 * EPS recommended 1e-9 on double, 1e-12 on long double
 * expected n ~ 100, 10ms. worst case is exponential */
```

```
using real_t = double;
using VD = vector<real_t>;
using VVD = vector<VD>;
const real_t EPS = 1e-9;
```

```
struct Simplex{
    int m, n;
    vector<int> B, N;
    VVD D;
```

```
Simplex(const VVD& A, const VD& b, const VD &c)
    : m(b.size()), n(c.size()), N(n+1), B(m), D(m+2, VD(n+2)){
    for(int i=0; i<m; ++i) for(int j=0; j<n; ++j) D[i][j] = A[i][j];
    for(int i=0; i<m; ++i) B[i] = n+i, D[i][n] = -1, D[i][n+1] = b[i];
    for(int j=0; j<n; ++j) N[j] = j, D[m][j] = -c[j];
    N[n] = -1; D[m+1][n] = 1;
}

void Pivot(int r, int s) {
    real_t inv = 1/D[r][s];
    for(int i=0; i<m+2; ++i){
        for(int j=0; j<n+2; ++j){
            if(i != r && j != s) D[i][j] -= D[r][j] * D[i][s] * inv;
        }
    }
    for(int i=0; i<m+2; ++i) if(i != r) D[i][s] *= -inv;
    for(int j=0; j<n+2; ++j) if(j != s) D[r][j] *= inv;
    D[r][s] = inv; swap(B[r], N[s]);
}

bool Phase(bool p) {
    int x = m + p;
    while(true) {
        int s = -1;
        for(int j=0; j<n; ++j){
            if(!p && N[j] == -1) continue;
            if(s == -1 || D[x][j] < D[x][s]) s = j;
        }
        if(D[x][s] > -EPS) return true;
        int r = -1;
        for(int i=0; i<m; ++i){
            if(D[i][s] <= EPS) continue;
            if(r == -1 || D[i][n+1] / D[i][s] < D[r][n+1] / D[r][s]) r = i;
        }
        if(r == -1) return false;
        Pivot(r, s);
    }
}

real_t solve(VD &x) {
    int r = 0;
    for(int i=1; i<m; ++i) if(D[i][n+1] < D[r][n+1]) r=i;
    if(D[r][n+1] < -EPS) {
        Pivot(r, n);
        if(!Phase(1) || D[m+1][n+1] < -EPS) return -1/0.0;
        for(int i=0; i<m; ++i) if(B[i] == -1) {
            int s = min_element(D[i].begin(), D[i].end() - 1) - D[i].begin();
            Pivot(i, s);
        }
    }
    if(!Phase(0)) return 1/0.0;
    x = VD(n);
    for(int i=0; i<m; ++i) if(B[i] < n) x[B[i]] = D[i][n+1];
    return D[m][n+1];
}
};
```

6.5 Range Prime Counting

// CREDITS: https://github.com/stjepang/snippets/blob/master/count_primes.cpp
 // Primes up to 10^{12} can be counted in ~1 second.

```
const int MAXN = 1000005; // MAXN is the maximum value of sqrt(N) + 2
bool prime[MAXN];
int prec[MAXN];
vector<int> P;

void init() {
    prime[2] = true;
    for (int i = 3; i < MAXN; i += 2) prime[i] = true;
    for (int i = 3; i*i < MAXN; i += 2){
        if (prime[i]){
            for (int j = i*i; j < MAXN; j += i*i) prime[j] = false;
        }
    }
    for(int i=1; i<MAXN; i++){
        if (prime[i]) P.push_back(i);
        prec[i] = prec[i-1] + prime[i];
    }
}

lint rec(lint N, int K) {
    if (N <= 1 || K < 0) return 0;
    if (N <= P[K]) return N-1;
    if (N < MAXN && 1ll * P[K]*P[K] > N) return N-1 - prec[N] + prec[P[K]];
    const int LIM = 250;
    static int memo[LIM*LIM][LIM];
    bool ok = N < LIM*LIM;
    if (ok && memo[N][K]) return memo[N][K];
    lint ret = N/P[K] - rec(N/P[K], K-1) + rec(N, K-1);
    if (ok) memo[N][K] = ret;
    return ret;
}

lint count_primes(lint N) { // less than or equal to
    if (N < MAXN) return prec[N];
    int K = prec[(int)sqrt(N) + 1];
    return N-1 - rec(N, K) + prec[P[K]];
}
```

6.6 Discrete Kth root

/* CREDITS:
<https://github.com/tzupengwang/PECaveros/blob/master/codebook/math/DiscreteKthsqrt.cpp>
 * Solve x for $x^P = A \bmod Q$
 * $(P, Q-1) = 1 \rightarrow P^{-1} \bmod (Q-1)$ exists
 * x has solution iff $A^{((Q-1)/P)} = 1 \bmod Q$
 * $PP \mid (Q-1) \rightarrow P < \sqrt{Q}$, solve $\lg Q$ rounds of discrete log
 * else \rightarrow find a s.t. $s \mid (Pa - 1) \rightarrow \text{ans} = A^a$ */
 using LL = long long;
 LL mul(LL x, LL y, LL mod){ return (__int128) x * y % mod; }
 LL add(LL x, LL y, LL mod){ return (x + y) % mod; }
 LL pw(LL x, LL y, LL mod){
 LL ret = 1, piv = x;

```
while(y){
    if(y & 1) ret = mul(ret, piv, mod);
    piv = mul(piv, piv, mod);
    y >>= 1;
}
return ret % mod;
}

void gcd(LL a, LL b, LL &x, LL &y, LL &g){
    if (b == 0) {
        x = 1, y = 0, g = a;
        return;
    }
    LL tx, ty;
    gcd(b, a%b, tx, ty, g);
    x = ty; y = tx - ty * (a / b);
}

LL P, A, Q, g; //  $x^P = A \bmod Q$ 
const int X = 1e5;
LL base, ae[X], aXe[X], iaXe[X];
unordered_map<LL, LL> ht;
#define FOR(i, c) for (int i = 0; i < (c); ++i)
#define REP(i, l, r) for (int i = (l); i <= (r); ++i)
void build(LL a) { // ord(a) = P < sqrt(Q)
    base = a;
    ht.clear();
    ae[0] = 1; ae[1] = a; aXe[0] = 1; aXe[1] = pw(a, X, Q);
    iaXe[0] = 1; iaXe[1] = pw(aXe[1], Q-2, Q);
    REP(i, 2, X-1) {
        ae[i] = mul(ae[i-1], ae[1], Q);
        aXe[i] = mul(aXe[i-1], aXe[1], Q);
        iaXe[i] = mul(iaXe[i-1], iaXe[1], Q);
    }
    FOR(i, X) ht[ae[i]] = i;
}

LL dis_log(LL x) {
    FOR(i, X) {
        LL iaXi = iaXe[i];
        LL rst = mul(x, iaXi, Q);
        if (ht.count(rst)) return i*X + ht[rst];
    }
}

LL main2() {
    cin >> P >> A >> Q;
    LL t = 0, s = Q-1;
    while (s % P == 0) {
        ++t;
        s /= P;
    }
    if (A == 0) return 0;
    if (t == 0) {
        //  $a^{P^{-1} \bmod \phi(Q)}$ 
        LL x, y, _;
        gcd(P, Q-1, x, y, _);
        if (x < 0) {
            x = (x % (Q-1) + Q-1) % (Q-1);
```

```

    }
    LL ans = pw(A, x, Q);
    if (pw(ans, P, Q) != A) while(1);
    return ans;
}
// A is not P-residue
if (pw(A, (Q-1) / P, Q) != 1) return -1;
for (g = 2; g < Q; ++g) {
    if (pw(g, (Q-1) / P, Q) != 1)
        break;
}
LL alpha = 0;
{
    LL y, _;
    gcd(P, s, alpha, y, _);
    if (alpha < 0) alpha = (alpha % (Q-1) + Q-1) % (Q-1);
}
if (t == 1) {
    LL ans = pw(A, alpha, Q);
    return ans;
}
LL a = pw(g, (Q-1) / P, Q);
build(a);
LL b = pw(A, add(mul(P%(Q-1), alpha, Q-1), Q-2, Q-1), Q);
LL c = pw(g, s, Q);
LL h = 1;
LL e = (Q-1) / s / P; // r^{t-1}
REP(i, 1, t-1) {
    e /= P;
    LL d = pw(b, e, Q);
    LL j = 0;
    if (d != 1) {
        j = -dis_log(d);
        if (j < 0) j = (j % (Q-1) + Q-1) % (Q-1);
    }
    b = mul(b, pw(c, mul(P%(Q-1), j, Q-1), Q), Q);
    h = mul(h, pw(c, j, Q), Q);
    c = pw(c, P, Q);
}
return mul(pw(A, alpha, Q), h, Q);
}

```

6.7 Miller-Rabin Test + Pollard Rho Factorization

```

namespace miller_rabin{
    lint mul(lint x, lint y, lint mod){ return (__int128) x * y % mod; }
    lint ipow(lint x, lint y, lint p){
        lint ret = 1, piv = x % p;
        while(y){
            if(y&1) ret = mul(ret, piv, p);
            piv = mul(piv, piv, p);
            y >>= 1;
        }
        return ret;
    }
}
bool miller_rabin(lint x, lint a){

```

```

    if(x % a == 0) return 0;
    lint d = x - 1;
    while(1){
        lint tmp = ipow(a, d, x);
        if(d&1) return (tmp != 1 && tmp != x-1);
        else if(tmp == x-1) return 0;
        d >>= 1;
    }
}
bool isprime(lint x){
    for(auto &i : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}){
        if(x == i) return 1;
        if(x > 40 && miller_rabin(x, i)) return 0;
    }
    if(x <= 40) return 0;
    return 1;
}

namespace pollard_rho{
    lint f(lint x, lint n, lint c){
        return (c + miller_rabin::mul(x, x, n)) % n;
    }
    void rec(lint n, vector<lint> &v){
        if(n == 1) return;
        if(n % 2 == 0){
            v.push_back(2);
            rec(n/2, v);
            return;
        }
        if(miller_rabin::isprime(n)){
            v.push_back(n);
            return;
        }
    }
    lint a, b, c;
    while(1){
        a = rand() % (n-2) + 2;
        b = a;
        c = rand() % 20 + 1;
        do{
            a = f(a, n, c);
            b = f(f(b, n, c), n, c);
        }while(gcd(abs(a-b), n) == 1);
        if(a != b) break;
    }
    lint x = gcd(abs(a-b), n);
    rec(x, v);
    rec(n/x, v);
}
vector<lint> factorize(lint n){
    vector<lint> ret;
    rec(n, ret);
    sort(ret.begin(), ret.end());
    return ret;
}

```

};

6.8 Highly Composite Numbers, Large Prime

< 10 ^k	number	divisors	2 3 5 7 11 13 17 19 23 29 31 37
1	6	4	1 1
2	60	12	2 1 1
3	840	32	3 1 1 1
4	7560	64	3 3 1 1
5	83160	128	3 3 1 1 1
6	720720	240	4 2 1 1 1 1
7	8648640	448	6 3 1 1 1 1
8	73513440	768	5 3 1 1 1 1 1
9	735134400	1344	6 3 2 1 1 1 1
10	6983776800	2304	5 3 2 1 1 1 1 1
11	97772875200	4032	6 3 2 2 1 1 1 1
12	963761198400	6720	6 4 2 1 1 1 1 1 1
13	9316358251200	10752	6 3 2 1 1 1 1 1 1 1
14	97821761637600	17280	5 4 2 2 1 1 1 1 1 1
15	866421317361600	26880	6 4 2 1 1 1 1 1 1 1 1
16	8086598962041600	41472	8 3 2 2 1 1 1 1 1 1 1
17	74801040398884800	64512	6 3 2 2 1 1 1 1 1 1 1 1
18	897612484786617600	103680	8 4 2 2 1 1 1 1 1 1 1 1 1

< 10 ^k	prime	# of prime	< 10 ^k	prime
1	7	4	10	9999999967
2	97	25	11	9999999977
3	997	168	12	99999999989
4	9973	1229	13	999999999971
5	99991	9592	14	9999999999973
6	999983	78498	15	99999999999989
7	9999991	664579	16	999999999999937
8	99999989	5761455	17	9999999999999997
9	999999937	50847534	18	99999999999999989

NTT Prime:

998244353 = 119 × 2²³ + 1. Primitive root: 3.985661441 = 235 × 2²² + 1. Primitive root: 3.1012924417 = 483 × 2²¹ + 1. Primitive root: 5.

7 Miscellaneous

7.1 Popular Optimization Technique

- Add or remove the constraints / Simplify the problem / Solve similar problem
- "Colliding ants problem" : Find a better way to describe the situation
- CHT. DnC optimization. Mo's algorithm trick (on tree). IOI 2016 Aliens trick. IOI 2009 Regions trick.
- Knuth's $O(n^2)$ Optimal BST : minimize $D_{i,j} = \min_{i \leq k < j} (D_{i,k} + D_{k+1,j}) + C_{i,j}$. Quadrangle Inequality : $C_{a,c} + C_{b,d} \leq C_{a,d} + C_{b,c}$, $C_{b,c} \leq C_{a,d}$. Now monotonicity holds.
- Sqrt batch processing - Save queries in buffer, and update in every sqrt steps (cf : IOI 2011 Elephant. hyea calls it "ainta technique")

- Dynamic insertion in static set (Make $O(\lg n)$ copy. Merge like binomial heap.)
- Offline insertion / deletion in insert-only set (Pair insertion-deletion operation, and regard it as range query)
- Atcoder Median Pyramid : Reduce the input to binary, and solve the easier problem.

7.2 Bit Twiddling Hack

```
int __builtin_clz(int x); // number of leading zero
int __builtin_ctz(int x); // number of trailing zero
int __builtin_clzll(long long x); // number of leading zero
int __builtin_ctzll(long long x); // number of trailing zero
int __builtin_popcount(int x); // number of 1-bits in x
int __builtin_popcountll(long long x); // number of 1-bits in x

lsb(n): (n & -n); // last bit (smallest)
floor(log2(n)): 31 - __builtin_clz(n | 1);
floor(log2(n)): 63 - __builtin_clzll(n | 1);

// compute next perm. ex) 00111, 01011, 01101, 01110, 10011, 10101..
long long next_perm(long long v){
    long long t = v | (v-1);
    return (t + 1) | (((~t & -~t) - 1) >> (__builtin_ctz(v) + 1));
}
```

7.3 Fast Integer IO

```
static char buf[1 << 19]; // size : any number geq than 1024
static int idx = 0;
static int bytes = 0;
static inline int _read() {
    if (!bytes || idx == bytes) {
        bytes = (int)fread(buf, sizeof(buf[0]), sizeof(buf), stdin);
        idx = 0;
    }
    return buf[idx++];
}
static inline int _readInt() {
    int x = 0, s = 1;
    int c = _read();
    while (c <= 32) c = _read();
    if (c == '-') s = -1, c = _read();
    while (c > 32) x = 10 * x + (c - '0'), c = _read();
    if (s < 0) x = -x;
    return x;
}
```

7.4 OSRank in g++

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

typedef
tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update> ordered_set;
```

```
ordered_set X;
X.insert(1); X.insert(2); X.insert(4); X.insert(8); X.insert(16);
```

```
cout<<*X.find_by_order(1)<<endl; // 2
cout<<*X.find_by_order(2)<<endl; // 4
cout<<*X.find_by_order(4)<<endl; // 16
cout<<(end(X)==X.find_by_order(6))<<endl; // true
```

```
cout<<X.order_of_key(-5)<<endl; // 0
cout<<X.order_of_key(1)<<endl; // 0
cout<<X.order_of_key(3)<<endl; // 2
cout<<X.order_of_key(4)<<endl; // 2
cout<<X.order_of_key(400)<<endl; // 5
```

7.5 Nasty Stack Hacks

```
// 64bit ver.
int main2(){ return 0; }
int main(){
    size_t sz = 1<<29; // 512MB
    void* newstack = malloc(sz);
    void* sp_dest = newstack + sz - sizeof(void*);
    asm __volatile__("movq %0, %%rax\n\t"
"movq %%rsp, (%%rax)\n\t"
"movq %0, %%rsp\n\t": : "r"(sp_dest): );
    main2();
    asm __volatile__("pop %%rsp\n\t");
    return 0;
}
```

7.6 C++ / Environment Overview

```
// vimrc : set nu sc ci si ai sw=4 ts=4 bs=2 mouse=a syntax on
```

```
// compile : g++ -o PROB PROB.cpp -std=c++11 -Wall -O2
// options : -fsanitize=address -Wfatal-errors
```

```
#include <bits/stdc++.h> // magic header
using namespace std; // magic namespace
```

```
struct StupidGCCCantEvenCompileThisSimpleCode{
    pair<int, int> array[1000000];
}; // https://gcc.gnu.org/bugzilla/show_bug.cgi?id=68203
```

```
// how to use rand (in 2017)
mt19937 rng(0x14004);
int randint(int lb, int ub){ return uniform_int_distribution<int>(lb, ub)(rng); }
```

```
// comparator overload
auto cmp = [](seg a, seg b){ return a.func() < b.func(); };
set<seg, decltype(cmp)> s(cmp);
map<seg, int, decltype(cmp)> mp(cmp);
priority_queue<seg, vector<seg>, decltype(cmp)> pq(cmp); // max heap
```

```
// hash func overload
```

```
struct point{
    int x, y;
    bool operator==(const point &p)const{ return x == p.x && y == p.y; }
};
struct hasher {
    size_t operator()(const point &p)const{ return p.x * 2 + p.y * 3; }
};
unordered_map<point, int, hasher> hsh;
```